arXiv:hep-ph/9411427v4 22 Dec 1994

The current of fermions scattered off a bubble wall

S.Yu. Khlebnikov[*]

Department of Physics, Purdue University, West Lafayette, IN 47907, USA

Abstract

Proceeding from WKB quantization conditions, we derive a semiclassical expression for the current of fermions scattered off a propagating bubble wall in the presence of longitudinal gauge field. It agrees with the expression used by Nasser and Turok in semiclassical analysis of instability of electroweak bubble walls with respect to longitudinal Z condensation. We discuss the resulting dispersion relation for longitudinal Z field and show that light species are important for the analysis of stability, because of their large contribution to plasma frequency.

---

1 Introduction

In a recent preprint [1], Nasser and Turok have found that interactions of bubble walls propagating during a first order electroweak phase transition with fermions can lead to a CP violating instability on the walls, a condensate of longitudinal Z bosons. This result, if confirmed by comprehensive study, is very important because it bears upon the main question in the theory of electroweak baryogenesis, namely, whether successful electroweak baryogenesis requires physics beyond the standard model or it does not. Because of deviations from equilibrium that occur during the wall propagation, the fermionic current, which enters the equation of motion for the Z field and contributes to its effective mass, cannot be obtained from thermodynamic relations and should be computed by direct averaging of a microscopic expression.

As a first approach to the problem, Nasser and Turok considered the WKB approximation for fermions and the thin-wall limit. (They have also checked results against a fully quantum-mechanical solution.) The expression for the current density they used in the WKB case would be obviously correct if the local value of canonical momentum of fermion remained unchanged during adiabatic switching on of the Z field. For a non-uniform Higgs field (bubble wall), this is not so, hence, a justification for that expression is required. In this note we shall show how that expression can be derived from WKB quantization conditions for fermions via certain manipulations with partial derivatives, similar to those used in thermodynamics. We shall also discuss the resulting dispersion relation for longitudinal Z field and show that, in realistic case, not only the top quark but also light species are important for the analysis of stability, because of their large contribution to plasma frequency.

2 Derivation of the formula for the current

The microscopic expression for the current density of a given species in the thin-wall case is

$$J(\mathbf{x}) = \sum_n f_n j_n(\mathbf{x}) \,, \tag{1}$$

where j_n is the current density for the n-th single-particle mode and f_n is the corresponding filling factor. The use of single-particle modes is specific of the thin-wall approximation that assumes that particles do not interact with each other while scattering off the wall. Though for a planar wall the non-trivial coordinate dependence is

only that for the direction orthogonal to the wall, we keep the full three-dimensional coordinate in (1) because some of the expressions below are more general than that special situation.

The partial current densities j_n are found by averaging the current density operator over eigenfunctions of the corresponding Dirac equation. If the Dirac hamiltonian is H, the current operator is

$$j(\mathbf{x}) = -\frac{\delta H}{\delta Z(\mathbf{x})} . \tag{2}$$

where Z is some component of the field (in our case, that orthogonal to the wall). The eigenfunctions ψ_n satisfy

$$H\psi_n = E_n \psi_n . \tag{3}$$

By a variant of the Feynman-Hellmann theorem (see for example ref.[2])

$$j_n(\mathbf{x}) = \psi_n^* j(\mathbf{x}) \psi_n = -\frac{\delta E_n}{\delta Z(\mathbf{x})} . \tag{4}$$

Hence, the partial currents are related to adiabatic, fixed n variations of the energy eigenvalues with respect to the field.

We can transform the fixed n variations to fixed energy variations via

$$\frac{\delta E_n}{\delta Z(\mathbf{x})} = -\left(\frac{\partial E_n}{\partial n}\right)_Z \left(\frac{\delta n}{\delta Z(\mathbf{x})}\right)_E , \tag{5}$$

which is derived by using jacobians (cf. ref.[3])

$$\frac{D(E,n)}{D(Z,n)} = \frac{D(E,n)}{D(E,Z)} \frac{D(E,Z)}{D(Z,n)} . \tag{6}$$

The resulting expression for the current is quite general (once the thin-wall limit is assumed) but it is especially useful in the WKB approximation, because in that case the quantum numbers have simple representation through WKB quantization conditions.

Like ref.[1], we assume the potential barrier created for particles by the wall and the Z field together to be monotonic. The WKB quantization conditions in the direction orthogonal to the wall (the only non-trivial ones) are

$$\int_{-L_z/2}^{L_z/2} p_z(z) dz = 2\pi n_z , \tag{7}$$

$$2\int_{-L_z/2}^{z^*(n)} p_z(z) dz = 2\pi n_z + C \tag{8}$$

3

for transmitted and reflected fermions, respectively. Here $p_z(z)$ is the longitudinal component of fermion momentum at a given spatial point, as determined from the classical dispersion law by conservation of single-particle energy and transverse momentum. n_z is a positive or negative integer. The classical turning point z^* of reflected particles depends on state $n = (n_x, n_y, n_z)$. A constant (up to exponential in $-L_z$ terms) phase shift C depends on the boundary condition for reflected particles at $-L_z/2$. The dispersion laws are listed in ref.[1].

A single-particle mode can be specified by the value of momentum at one of the infinities (more precisely, at $\pm L_z/2$), $\mathbf{p}_\infty$. That incident momentum is related to the energy of the mode by the usual, field-independent expression. So, at given $p_{x,\infty}$, $p_{y,\infty}$, the variation at fixed energy is variation at fixed $p_{z,\infty}$.

Next, we use n_z instead of n in (5). Going over to one-dimensional variations we obtain

$$\frac{\delta E_n}{\delta Z(z)} = -\frac{1}{L_x L_y}\left(\frac{\partial E_n}{\partial n_z}\right)_Z \left(\frac{\delta n_z}{\delta Z(z)}\right)_{p_{z,\infty}} = -\frac{\eta}{(2\pi)L_x L_y}\left(\frac{\partial E_n}{\partial n_z}\right)_Z \left(\frac{\partial p_z(z)}{\partial Z(z)}\right)_{p_{z,\infty}} \quad (9)$$

where η is 1 for transmitted states, 0 for reflected states with $z^* < z$, and 2 for reflected states with $z^* > z$. Note that the variational derivative with respect to Z is converted into ordinary one. The derivatives are taken at fixed $p_{x,\infty}$, $p_{y,\infty}$, which is not indicated explicitly in (9).

Using (9) and another transformation of derivatives

$$\left(\frac{\partial p_z(z)}{\partial Z(z)}\right)_{p_{z,\infty}} = -\left(\frac{\partial p_z(z)}{\partial p_{z,\infty}}\right)_Z \left(\frac{\partial p_{z,\infty}}{\partial Z}\right)_{p_z}, \quad (10)$$

we transform the current density as follows

$$\begin{aligned}
J(z) &= \int dn_x dn_y dn_z f_n j_n(z) \\
&= -\int \eta \frac{dp_{x,\infty} dp_{y,\infty}}{(2\pi)^3} dn_z f(\mathbf{p}_\infty)\left(\frac{\partial E_n}{\partial n_z}\right)_Z \left(\frac{\partial p_z(z)}{\partial p_{z,\infty}}\right)_Z \left(\frac{\partial p_{z,\infty}}{\partial Z}\right)_{p_z} \\
&= -\int \eta \frac{dp_{x,\infty} dp_{y,\infty}}{(2\pi)^3} dn_z f(\mathbf{p}_\infty)\frac{\partial E}{\partial p_{z,\infty}}\left(\frac{\partial p_{z,\infty}}{\partial n_z}\right)_Z \left(\frac{\partial p_z(z)}{\partial p_{z,\infty}}\right)_Z \left(\frac{\partial p_{z,\infty}}{\partial Z}\right)_{p_z} \\
&= -\int \eta \frac{d^3 p_\infty}{(2\pi)^3} f(\mathbf{p}_\infty)\left(\frac{\partial p_z(z)}{\partial p_{z,\infty}}\right)_Z \left(\frac{\partial E}{\partial Z}\right)_{p_z}. \quad (11)
\end{aligned}$$

The last line is the expression used in ref.[1]. We have thus shown how it can be derived from WKB quantization conditions.

3   Dispersion relation and plasma frequency

The formula for the current allows us to compute the response of the non-equilibrium plasma to longitudinal Z field. Because in the WKB calculation of the current both the wall profile and the gauge field were assumed slowly varying, we are actually considering the zero-momentum limit of the response. Accordingly, the relation (11) between the current and the field is local in space.

The linearized equation of motion — the dispersion relation for the Z field is

$$\text{diag}(\omega^2 - m_Z^2(z),\ \omega^2) - \hat{\omega}_p^2(z) = 0 \ . \tag{12}$$

This is a matrix equation because of the mixing between Z and photon field A due to plasma effects; $\hat{\omega}_p^2(z)$ is the non-equilibrium plasma frequency matrix,

$$\hat{\omega}_p^2(z) = - \left(\begin{array}{cc} \partial J(z)/\partial Z(z) & \partial J(z)/\partial A(z) \\ \partial J_{em}(z)/\partial Z(z) & \partial J_{em}(z)/\partial A(z) \end{array} \right)_{Z=A=0} \ , \tag{13}$$

where J now denotes the total, summed over all species, current coupled to Z and J_{em} is the total electromagnetic current.

Non-equilibrium effects in the plasma frequency are important only for the heaviest fermion, the top quark, which interacts effectively with the bubble wall. Evaluation of the integral (11) shows [1] that for non-zero wall velocity, fermions produce a contribution to the current which has a square-root singularity in $m_\infty - m(z)$, where m_∞ is the mass of a fermion in the broken phase at infinity and $m(z)$ is its mass locally (so the singularity is behind the wall). This singular term corresponds to a negative contribution to the upper-left entry of the plasma frequency matrix,

$$\Delta\omega_{p11}^2(z) \sim -\frac{\alpha_W}{4\pi \cos^2\theta_W} \frac{u m_\infty^3}{[m_\infty^2 - m^2(z)]^{1/2}} \ , \tag{14}$$

where $\alpha_W = g^2/(4\pi)$ is the weak interaction constant, u is the wall velocity. One can see that the typical value of $p_z(z)$ that give rise to the singular term (14) is of order $[m_\infty - m(z)]^{1/2}$. For the WKB approximation to be applicable, this should be large compared to the inverse thickness of the wall given roughly by the Higgs mass m_H (all masses are those at the phase transition temperature). So, within the WKB domain, the singularity provides an enhancement factor of order

$$m_\infty/[m_\infty^2 - m^2(z)]^{1/2} \lesssim m_\infty/m_H \ , \tag{15}$$

which can be considerable for top quarks.

However, to decide if non-equilibrium contribution of top quarks to the dispersion relation, whether obtained by WKB or fully quantum-mechanical means, leads to an instability of the $Z = 0$ state, we should consider it together with contributions of other particles.[†] In particular, numerous light species (those with masses much smaller than temperature) give an essentially equilibrium contribution to $\hat{\omega}_p^2$. The equilibrium contribution of fermions and *transverse* W bosons to $\hat{\omega}_p^2$ can be obtained simply by orthogonal transformation of the diagonal matrix of plasma frequencies of the $SU(2) \times U(1)$ basis. With N_g light fermionic generations, light transverse W, and small wall velocity, this contribution is

$$(\hat{\omega}_p^2)_{eq} \approx \frac{g^2 T^2}{9c^2} \begin{pmatrix} N_g \left(1 - 2s^2 + \frac{8}{3}s^4\right) + 2c^4 & N_g \left(1 - \frac{8}{3}s^2\right) sc + 2sc^3 \\ N_g \left(1 - \frac{8}{3}s^2\right) sc + 2sc^3 & \left(\frac{8}{3}N_g + 2\right) s^2 c^2 \end{pmatrix} , \qquad (16)$$

where $c \equiv \cos\theta_W$, $s \equiv \sin\theta_W$. For our estimates, we took $N_g = 3$ but subtracted the top-quark contribution from (16), considered W bosons as light, and neglected the role of longitudinal W and Higgs bosons.

In the WKB approximation, the quantity of interest is the magnitude of the non-equilibrium top-quark contribution to $\hat{\omega}_p^2$, the most significant part of which is (14), corresponding to the onset of instability. It is determined approximately from

$$\det \begin{pmatrix} \Delta\omega_{p11}^2(z) + m_Z^2(z) + A & B \\ B & C \end{pmatrix} = 0 , \qquad (17)$$

where A, B, C are the entries of the equilibrium plasma frequency matrix of light species. (In a fully quantum-mechanical, non-WKB calculation, the relation between the current and the field becomes non-local [1] and so does the criterion for instability.) Estimating the plasma frequency of light species as described above, we find from (17) that the onset of instability is at

$$-\Delta\omega_{p11}^2(z) = m_Z^2(z) + 0.25 g^2 T^2 / \cos^2\theta_W . \qquad (18)$$

For realistic values of ϕ_∞/T, where ϕ_∞ is the expectation value of the Higgs field in the broken phase, the second term on the right-hand side is of the same order as m_Z^2 in the broken phase, for example, for $\phi_\infty = 1.5T$ used in ref.[1], about 45% of it.

We conclude that light species are in general important for the analysis of stability, because of their large contribution to the dispersion relation.

References

[1] S. Nasser and N. Turok, Princeton preprint PUPT-94-1456 (1994), hep-ph/9406270.

[2] D.J. Griffiths, *Introduction to Quantum Mechanics*, Prentice Hall, 1994; Problem 6.27.

[3] L.D. Landau and E.M. Lifshitz, *Statistical Physics*, Part 1, Pergamon, 1980; Sect.16.